Model Predictive Control for Trajectory Tracking on Crazyflie 2.1 Quadrotor

Authors: Samuel Preston Jones, Nicolas Cheptine Smith {s.preston.jones, n.smith.2}@student.rug.nl

Integration Project. Bernoulli Institute for Mathematics, Computer Science and Artificial Intelligence, University of Groningen

Supervisor: Kailai Li

Abstract

This work presents the design, implementation, and experimental validation of a Model Predictive Control (MPC) architecture for trajectory tracking on the Crazyflie 2.1 nano-quadrotor. A Separated Guidance and Control (SGC) scheme delegates fast attitude stabilization to the onboard PID controller operating at 500 Hz, thereby reducing the dimensionality of the optimization. The OCP is solved at 100 Hz using acados with CasADi, with state feedback from a motion capture system. The controller is validated on helix and lemniscate trajectories. For the lemniscate, MPC achieved position RMSEs of 2.2 cm, 1.8 cm, and 1.7 cm, while for the helix, it achieved position RMSEs of 1.9 cm, 3.8 cm, and 1.9 cm in x, y, and z, respectively.

Background

UAVs are attracting growing interest across research and industry due to their versatility in diverse applications such as agriculture and search and rescue. Quadrotors are underactuated; they have 4 motors but 6 DoF. This leads to highly coupled dynamics, which pose a considerable challenge for autonomous flight control. MPC offers a principled framework for this challenge by explicitly accounting for system dynamics, anticipating future behavior, enforcing state and input constraints directly within the control law, and robustly adapting to disturbances by continuously resolving an optimal control problem.

System Architecture

Motivated by the onboard attitude controller, a Separated Guidance and Control architecture was adopted. The MPC outer loop generates attitude and thrust commands; fast attitude stabilization is delegated to the onboard PID controller running at 500 Hz. This reduces the dimensionality of the optimization and makes real-time off-board MPC feasible.

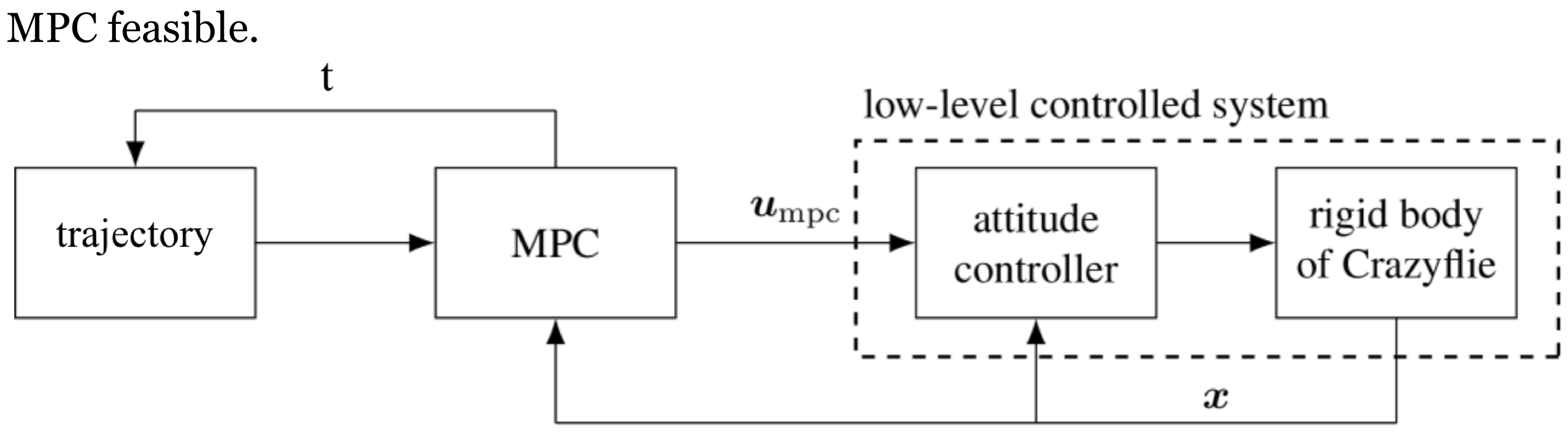

Figure 1: SGC system architecture.

Quadrotor Model

The system state is

$$x = [\xi^\top,\ \dot{\xi}^\top,\ \eta^\top]^\top \in \mathcal{X} \subset \mathbb{R}^9,$$

where $\xi = [x,\ y,\ z]^\top$ denotes position in the inertial frame and $\eta = [\phi,\ \theta,\ \psi]^\top$ represents the Euler angles (roll, pitch, yaw). The MPC control input is

$$u = [\Delta T,\ \phi_{\text{cmd}},\ \theta_{\text{cmd}},\ \dot{\psi}_{\text{cmd}}]^\top \in \mathcal{U} \subset \mathbb{R}^4.$$

The translational dynamics follow from Newton's equations, while the attitude dynamics can be approximated by a first-order response due to the inner loop stabilizer:

$$\ddot{\xi} = -\begin{bmatrix}0\\0\\g\end{bmatrix} + \frac{1}{m} R_{IB}(\eta)\begin{bmatrix}0\\0\\\Delta T + mg\end{bmatrix}, \qquad \dot{\phi} = \frac{1}{T_\phi}(\phi_{\text{cmd}} - \phi), \quad \dot{\theta} = \frac{1}{T_\theta}(\theta_{\text{cmd}} - \theta), \quad \dot{\psi} = \dot{\psi}_{\text{cmd}}.$$

MPC Formulation

A reference trajectory $y_{\text{ref}}(t) = [x_{\text{ref}}(t),\ y_{\text{ref}}(t),\ z_{\text{ref}}(t),\ \psi_{\text{ref}}(t)]^\top$ is provided a priori. At each time t, the MPC solves the Optimal Control Problem (OCP):

$$\min_{\bar{u}(\cdot\,|\,t)} \int_t^{t+T_p} \|y(\tau) - y_{\text{ref}}(\tau)\|_Q^2 + \|\bar{u}(\tau\,|\,t)\|_R^2 \, \mathrm{d}\tau$$

subject to

$$\begin{aligned}
&\dot{\bar{x}}(\tau\,|\,t) = f(\bar{x}(\tau\,|\,t),\ \bar{u}(\tau\,|\,t)), \quad \forall \tau \in [t,\ t+T_p]\\
&\bar{x}(\tau\,|\,t) \in \mathcal{X}, \quad \bar{u}(\tau\,|\,t) \in \mathcal{U}, \quad \forall \tau \in [t,\ t+T_p]\\
&\bar{x}(t\,|\,t) = x(t)
\end{aligned}$$

where Q and R are positive definite weighting matrices, $\mathcal{X}$ and $\mathcal{U}$ encode state and input constraints, and T_p is the prediction horizon.

The OCP is solved using acados with CasADi, with a multiple shooting scheme and ERK4 discretization. The MPC is run at 100 Hz, and the previous solution is used as a warm start to reduce solving time.

The prediction horizon was set to $T_p = 1.5\,\text{s}$ with 50 shooting nodes, giving a discretization step of $30\,\text{ms}$, and the time constants were set to $T_\phi = T_\theta = 0.08\,\text{s}$. The weighting matrices were tuned as $Q = \text{diag}(12, 12, 12,\ 3, 3, 9,\ 0, 0, 1)$, and $R = \text{diag}(2, 2, 2, 5)$. Position tracking is weighted most heavily; vertical velocity is penalized more than lateral velocities to aid altitude regulation. Roll and pitch are excluded from the cost and handled via constraints instead. The thrust command carries the highest input penalty, which was found to suppress oscillatory behavior.

Experimental Setup

The controller runs off-board on a laptop, communicating with the Crazyflie 2.1 firmware via a Crazyradio USB dongle using the CrazySwarm2 ROS2 stack as middleware. The position and orientation estimates were obtained by combining the MoCap and IMU measurements using the onboard Kalman filter. Velocity was estimated via finite differences on the MoCap position signal, with the onboard Kalman estimate used as a fallback for anomalous readings.

The Crazyflie accepts PWM commands in $[0, 6000]$ while the MPC requires thrust in Newtons. A linear fit was used to perform this conversion; it yielded $R^2 = 0.9989$, and its inverse is applied at each control step to convert MPC thrust outputs to PWM commands.

Results & Conclusion

To evaluate the performance of the proposed control architecture, two reference trajectories were defined as smooth, parametric curves in three-dimensional space: a helix (for simultaneous 3-axis tracking) and a lemniscate (frequent curvature changes).

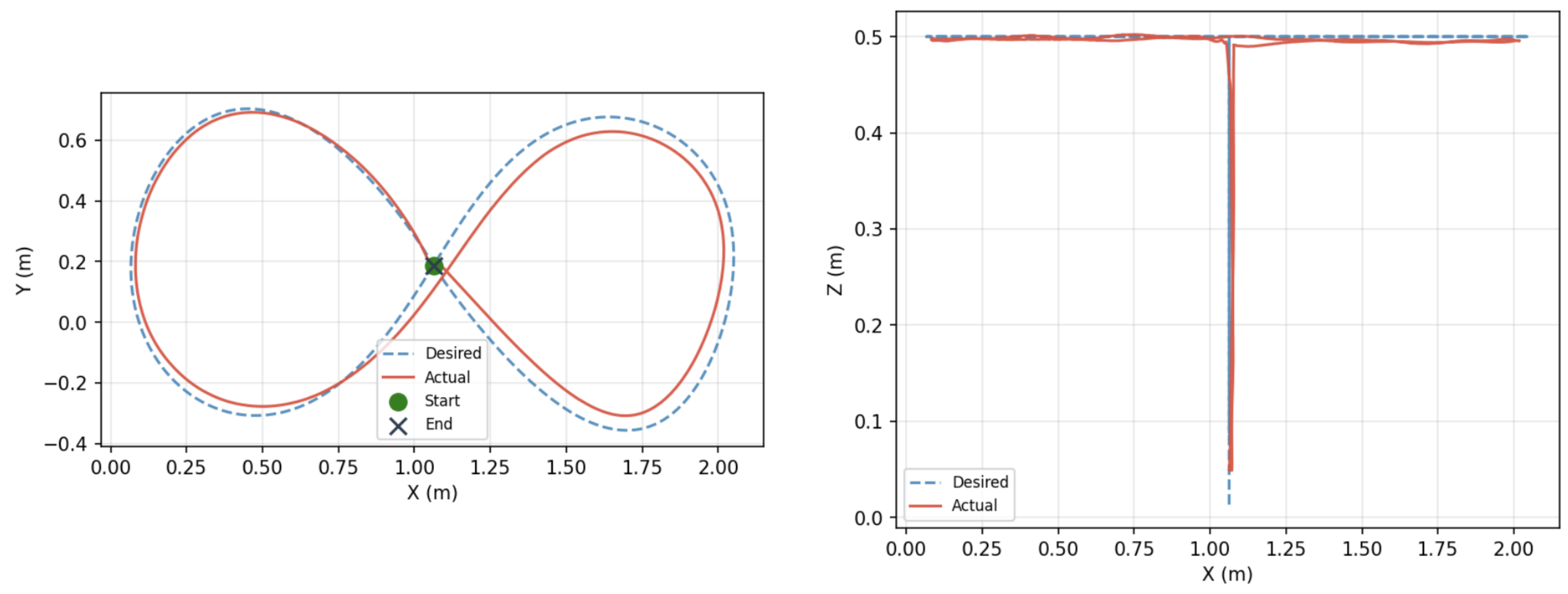

Figure 2: Trajectory tracking accuracy for lemniscate trajectory.

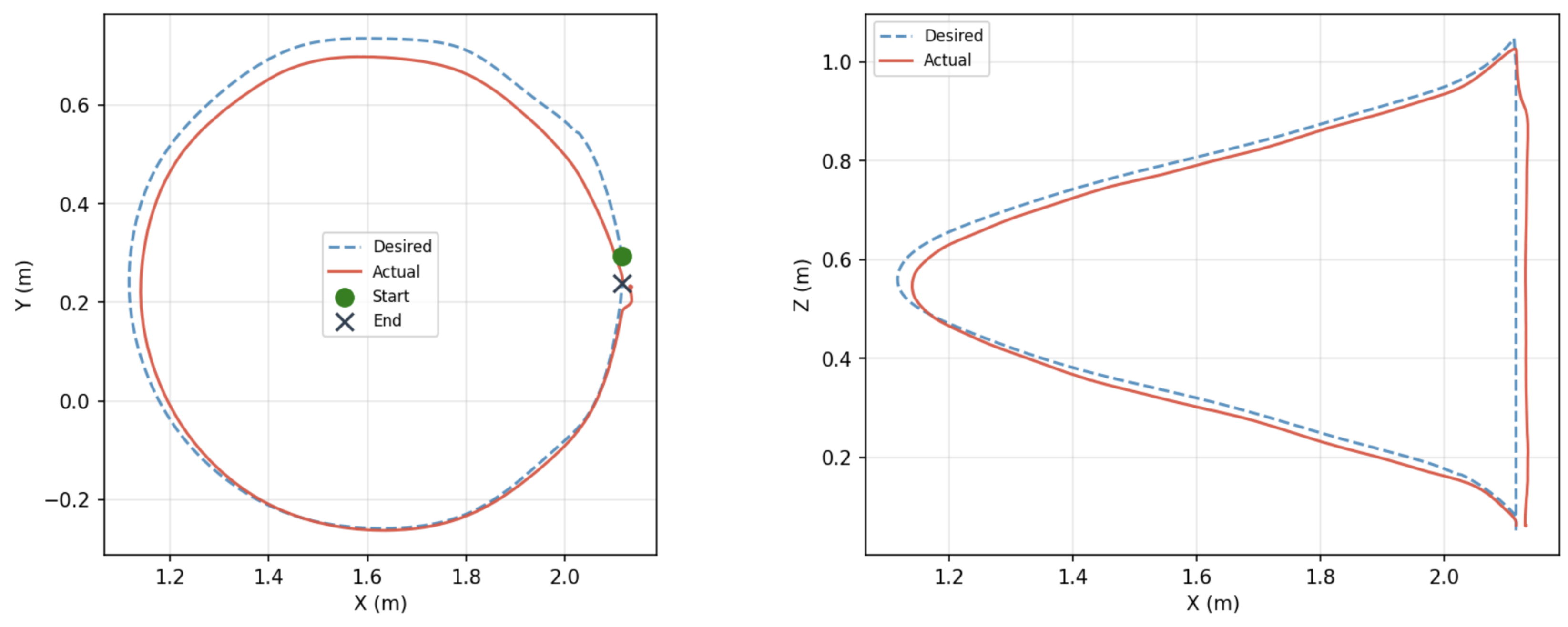

Figure 3: Trajectory tracking accuracy for helix trajectory.

For the lemniscate, MPC achieved position RMSEs of 2.2 cm, 1.8 cm, and 1.7 cm in x, y, and z, respectively. For the helix, MPC achieved position RMSEs of 1.9 cm, 3.8 cm, and 1.9 cm in x, y, and z, respectively.

Tracking errors are largest during the high-curvature segments, which require steeper changes in state, peaking at roughly 6 cm in the x-axis for the former trajectory and peaking at 13 cm in the y-axis for the latter. In both, the z-axis maintains near-perfect altitude tracking, confirming good height regulation.

Overall, the controller achieves consistent sub-4 cm RMSE across all axes, with errors driven primarily by transient dynamics rather than steady-state offsets.

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